Experiment to Detect Frame Dragging in a Lead Superconductor

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Abstract

Recent work by Tajmar and de Matos predicts a greatly enhanced gravitomagnetic field is measurable in the vicinity of a rotating superconductor. They predict that the associated frame dragging is measurable when the density of Cooper pairs is sufficiently large relative to the mass density. Experimental measurements with superconducting lead and niobium samples reported by the same group support this theory. We have conducted an experiment with superconducting lead and a very large ring laser gyroscope. No frame dragging effect was observed by us. We find that if the effect exists it is at least 22 times smaller than predicted by the theory, otherwise it would have been detected in our experiment with probability $\geq 95\%$.

Key words: Frame-dragging, Lense-Thirring, ring laser, gravitomagnetism, superconductor

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1 Introduction

The Lense-Thirring effect, also known as *inertial frame dragging*, is a predicted consequence of general relativity and is expected to occur in the vicinity of a rotating massive body. Lense and Thirring predicted that the rotation of a massive object would alter the space time around it, in effect causing nearby inertial frames to rotate slightly. For a spherical body with moment of inertia I, rotating at rate $\mathbf{\Omega}$, the induced inertial frame rotation $\mathbf{\Omega}'$ at distance $\mathbf{R}$ is given by

$$\mathbf{\Omega}' = \frac{GI}{c^2R^3}\left[\frac{3\mathbf{R}}{R^2}\left(\mathbf{\Omega}\cdot\mathbf{R}\right) - \mathbf{\Omega}\right] \tag{1}$$

where c is the speed of light and G the gravitational constant [1]. For the earth, the effect is extremely small, amounting to 4×10^{-14} rad/s at a geographic pole, that is of the order of one part in 10^9 of earth rotation.

The Gravity Probe B experiment, (currently nearing completion) intends to measure the Lense-Thirring field for the rotating Earth. The Canterbury ring laser group operates [1] the world's most precise ring laser gyros and has previously considered experiments to measure frame-dragging [2]. They concluded that due to the smallness of the classical effect, measurement is well beyond the reach of current instruments.

It is possible to linearize the equations of general relativity for small perturbations and weak fields, giving 3-vectors which obey equations analogous to Maxwell's equations for the electric and magnetic fields. The GEM equations [3, 4] of *gravitomagnetism* are an example of such a reformulation. In this context the Lense-Thirring result can be referred to as a *gravitomagnetic* effect.

Recently, Tajmar and de Matos [5,6] have proposed that under certain circumstances the gravitomagnetic field, B_g^*, of a rotating superconductive body may be very much larger than the classical field B_g that results from equation 1. Specifically, they predict a gravitational analogue of the London magnetic dipole field. The superconductor's rotation is coupled to the observed rotation of the surrounding space and is proportional to the London field. The magnitude of the gravitomagnetic field of a superconductor is

$$B_g^* = 2\omega \frac{\rho^*}{\rho} \tag{2}$$

where ρ is the classical mass density of the superconducting material and ρ^* is the mass density of Cooper pairs in the superconductor. They propose this effect as an explanation for the disagreement between theoretical and experimental values of the mass of Cooper pairs in niobium measured by Tate [7, 8].

This field, while still small enough that gross macroscopic frame-dragging effects are not immediately obvious, is enormously larger (by a factor of order 10^{30}) than the classical field. This opens the possibility for detection in a laboratory experiment. Recent experimental work to this end performed by Tajmar and de Matos [9] appears to support this. When superconducting lead and niobium rings were rotated, nearby linear accelerometers detected a transient during acceleration and again during deceleration. The effect disappeared when the temperature of the rings was increased above the superconducting transition.

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If this effect is as large as claimed and can be shown to survive independent verification then the impact on gravitational physics would be tremendous. It is interesting to note from equation 2 that this effect is not simply a larger Lense-Thirring field. It is fundamentally different because the magnitude of the effect does not depend on the mass of the superconductor. However from a practical point of view the inverse cube decay of a dipole field means that the observed field will be larger for superconductors with larger volume.

2 Predicted rotational coupling

A ring laser gyro measures rotation relative to an inertial frame of reference. The Canterbury ring laser gyro UG-2 [10,11] used in this experiment is rigidly attached to the Earth. This means we need to subtract the Sagnac signal due to constant rotation of the earth from the gyro's output. (The Sagnac signal is the difference in frequency between the clockwise and anti-clockwise laser beams.) It also means that there are various seismic effects observable on the Sagnac signal. These effects have been well studied by this group [1, 10] and over the short term typically correspond to about 1.5 parts per million of the constant earth rotation signal.

The gravitomagnetic field outside a spinning body is a dipole field, sketched in figure 1. A quantitative calculation of the vector field allows us to determine the rotation that would be sensed by a ring laser gyro at an arbitrary location relative to the spinning superconducting body. A ring laser gyro placed as in figure 2 in the equatorial plane of the spinning body would measure a rotation in the same direction as the rotation of the body.

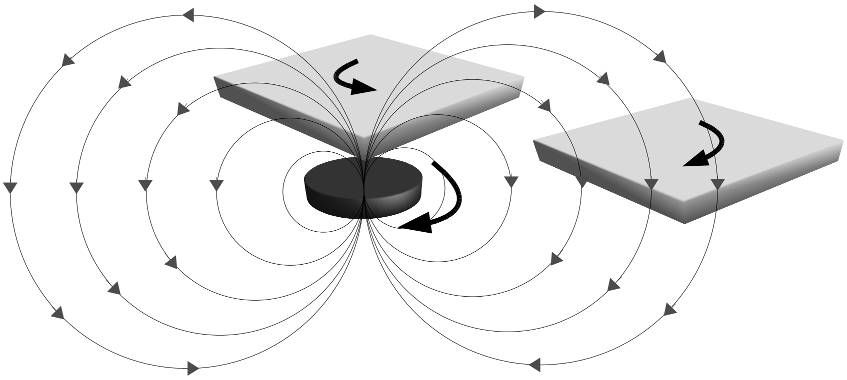

Fig. 1. Schematic of the coupling between the rotating superconducting body (dark cylinder) and nearby laser gyros (light squares). Thin lines approximate the gravitomagnetic field lines. Thick lines represent direction of rotation; *actual* rotation for the superconductor and *indicated* rotation for the laser gyros.

When in a gravitomagnetic field, a ring laser would measure a signal propor-

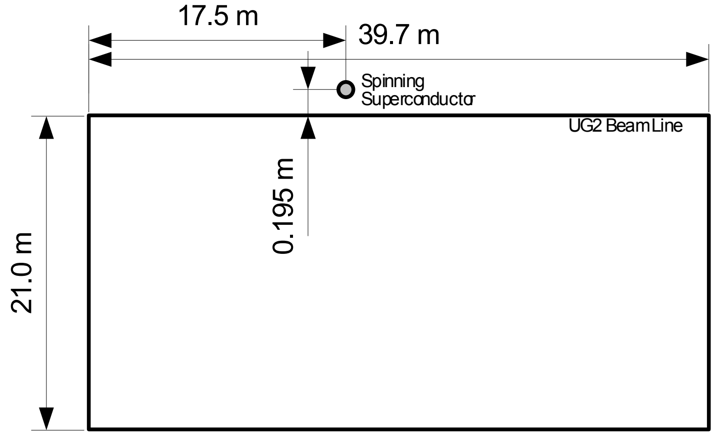

Fig. 2. Setup of the frame dragging experiment with the UG-2 laser.

tional to the superconductor rotation rate. With a linear accelerometer only a transient effect would be observable, and only when the superconductor is being rotationally accelerated or decelerated. From a practical standpoint a ring laser rotation sensor makes it much easier to decouple the measurement from any vibrational or mechanical coupling that might occur as the superconductor is undergoing angular acceleration.

3 Experimental apparatus

UG-2 [10, 11] is a 833.7 m^2 ring laser with nominal Sagnac frequency of 2176.785 Hz, and typical microseismic noise level of 3 mHz. The frame-dragging apparatus (the Dewar system with its spinning superconductor) has an overall size of 0.15 m by 0.15 m by 0.44 m and was isolated from the nearest corner component of the interferometer by some 17.5 m of rock.

The test apparatus (figure 3) consists of a small glass Dewar (110 mm internal diameter and silvered in its vacuum space) which contains liquid helium (LHe), helium vapour and the spinning superconducting body. This glass Dewar is supported entirely inside large-diameter rigid Perspex tube, which in turn is inside a larger stainless steel Dewar containing liquid nitrogen (LN2). The Perspex tube is sealed at the top of the inner Dewar and extends beyond the top of the outer Dewar. The tube ensures complete separation of the boil-off gases from the LHe and LN2. The tube is closed off with 19 mm thick Perspex windows at the top of inner Dewar and at its upper end. The windows allow visual monitoring of the equipment inside the inner Dewar. Extra holes in the windows allow access for a LHe transfer tube and a carbon-ceramic temperature sensor which was located just above the rotating superconductor. During operation unused holes in the upper window are plugged.

The superconducting rotor, a high-purity (99.9%) lead cylinder (topologically a ring with outside diameter 91 mm, inside diameter 8 mm, thickness 38 mm) is suspended through simple plastic bearings by a vertical glass-reinforced plastic axle (thermal conductivity $\sim 0.1\,\mathrm{Wm^{-1}K^{-1}}$ at liquid helium temperature [12]) passing through both windows. This allows the rotor to be spun by an external electric motor. Heat transfer through the axle is negligible.

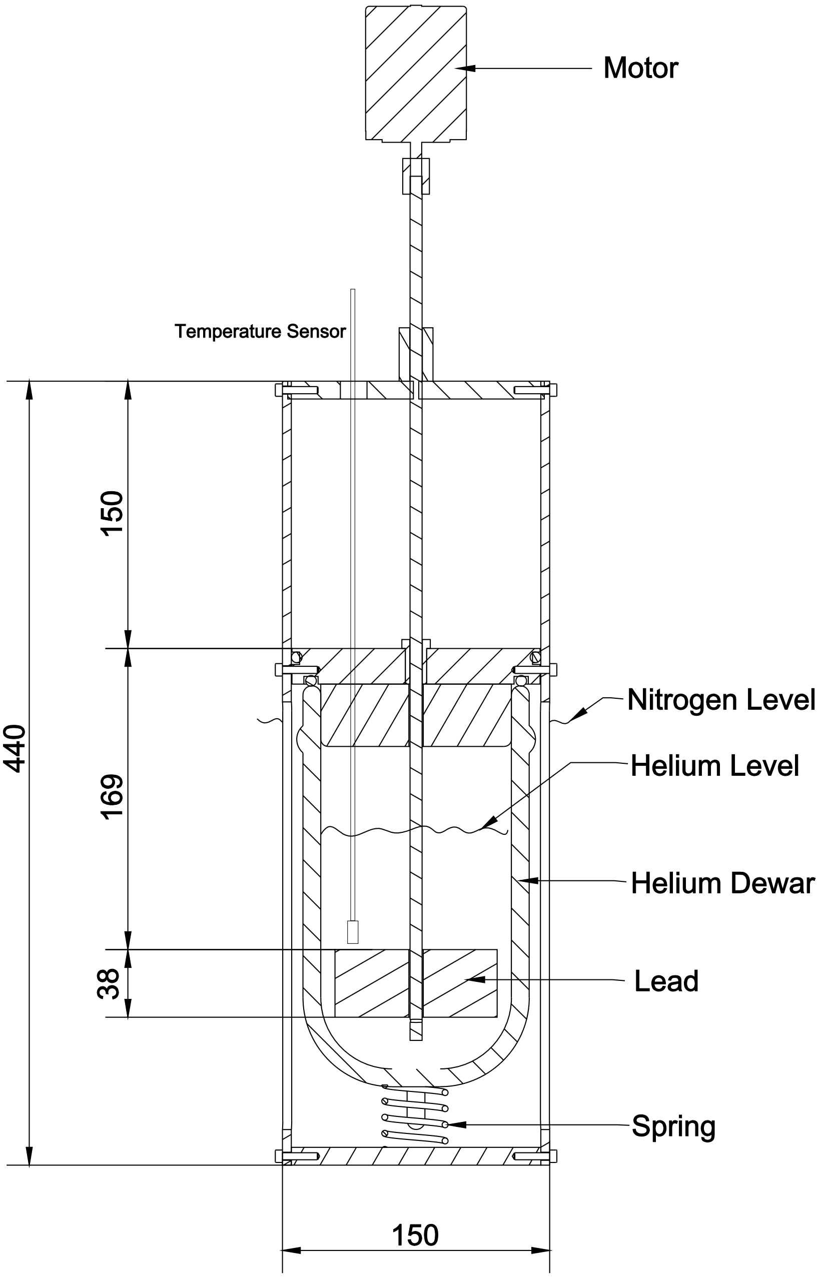

Fig. 3. Side-on plan of the apparatus (omitting the outer LN2 dewar). All measurements in mm.

4 Predicted field calculations

To evaluate the gravitomagnetic field of equation 2 we require the mass density ρ^* of Cooper pairs in the sample of superconducting lead. This follows from the number density, which may be calculated from the theoretical expression for the London penetration depth,

$$\lambda = \sqrt{\frac{m_s}{\mu_0 n_s q^2}} \tag{3}$$

where m_s is the mass of the Cooper pairs, n_s their number density, q their charge ($2e$) and $\mu_0 = 4\pi \times 10^{-7}$ H/m. We also require a measured value of λ. For our experiment at liquid helium temperature (4.22 K) the measured penetration depth for lead is $\lambda \approx 480$ Å [13]. With substitution of values, the mass density of Cooper pairs is given by:

$$\rho^* = n_s m_s = \frac{m_e^2}{\mu_0 \lambda^2 e^2} = 0.011 \quad \mathrm{kgm}^{-3} \tag{4}$$

The density of lead at LHe temperature $\rho \approx 11635$ kg m^{-3}. Thus for our sample,

$$B_g = 1.89 \times 10^{-6} \omega \tag{5}$$

This is comparable to the result of Tajmar *et al.* who quote a value of $3.9 \times 10^{-6}\omega$ for niobium at 0 K. The value calculated here pertains to the surface of the rotor. In order to calculate the far field we assume that the gravitomagnetic field is everywhere proportional to the London magnetic field.

We now calculate the effective rotation as observed by the UG-2 laser. The experiment was positioned so the rotor equatorial plane coincided with the plane of the laser, and the rotation axis was 0.195 m outside the beam path of the laser. The layout is shown in figure 2. It is sufficient to evaluate the mean value of the vertical component of the gravitomagnetic field of the rotor over the area enclosed by the beam path. Additional calculations show that at all distances greater than 0.195 m, to sufficient accuracy the London field and hence by assumption the gravitomagnetic field have an inverse cube dependency.

For our rotor, the vertical gravitomagnetic field strength at 1 m from the rotation axis is calculated as $9.15 \times 10^{-11}\omega$. A mean value, averaged over the area of the laser, then follows by numerical integration. We obtain $B_g' = 1.11 \times 10^{-12}\omega$. We see that for practical rotor speeds the predicted effect is rather small, but the remarkable sensitivity of the UG-2 laser gyro makes it readily measurable.

When an active laser gyro with area $\mathbf{A}$ rotates, the optical frequencies of the two output beams (as observed in the rotating frame) differ. The difference or *Sagnac frequency* f_s is proportional to the rotation rate $\mathbf{\Omega}$ in accordance with the Sagnac equation (equation 6), where the operating laser wavelength in the absence of rotation is λ and optical path length is P.

$$f_s = \frac{4\mathbf{A}.\mathbf{\Omega}}{\lambda P} \tag{6}$$

The contribution to the Sagnac frequency due to the total gravitomagnetic flux AB_g' intersecting the ring laser is then:

$$\Delta f_s = \frac{4AB_g'}{\lambda P} \tag{7}$$

In terms of our experimental parameters, the Tajmar and de Matos model gives the prediction that the observed change in Sagnac frequency is

$$\Delta f_s = 4.81 \times 10^{-5}\omega \quad \text{Hz} \tag{8}$$

5 Experiment and results

The experiment was completed successfully. The windows allowed the filling with LHe to be observed, and the rotor was seen to be fully immersed in LHe at the start and finish of the experiment. When the rotor was spinning, the LHe rose up the walls of the dewar and a turbulent condition arose in the LHe (partly due to disruption of the flow by the temperature sensor support). Liquid helium was in contact with the lead at all times. The boil-off rate increased somewhat during the experiment. The nearby temperature sensor indicated liquid helium temperature for the duration of the experiment. We are confident that the lead sample remained well below the transition temperature of 7.192 K and therefore was superconducting for the duration of the experiment.

The rotor was spun both clockwise and counterclockwise a number of times. The results used in the analysis below come from a run of 5 minutes clockwise followed by 5 minutes stationary followed by 5 minutes counterclockwise rotation (directions of rotation as viewed from above). The laser gave very good performance over this time and geophysical disturbances were minor. A Sagnac stability of 0.2 Sagnac cycles (relative to a GPS-locked reference generator set at 2176.785 Hz) was achieved over the period of measurement. An earlier control run done before the LHe was introduced showed no deviations greater than this.

The raw measurements of Sagnac frequency have significant quasiperiodic fluctuations, with periods of ∼5 s and ∼25 s, both of geophysical (microseismic) origin. In presenting our final result these effects have been filtered out using a zero phase delay (see Gustafsson 1996 [14]) 3rd order Butterworth low-pass digital filter with an upper passband frequency of 33.3 mHz (30 s period). Finally, the frequency deviation and its standard error have been calculated over each 30 s period.

The rotor speed was 15 revolutions s^{-1} in the clockwise direction and 12 revolutions s^{-1} in the counter-clockwise direction. These give expected Sagnac frequency differences of 4.38 mHz and 3.51 mHz respectively. We expect an increase in Sagnac frequency with clockwise rotation and a decrease with counterclockwise rotation.

Figure 4 shows a plot of the results. The solid horozontal lines indicate the average Sagnac deviation over each period of rotation. The 15 s either side of each transition in rotational velocity have not been included in this averaging.

Figure 5 shows a plot of the average Sagnac deviation from figure 4 plotted against rotational velocity of the superconductor. A weighted least squares fit to the data through the origin has been computed and is plotted as a solid line. The slope of this line is $(1.1 \pm 2.4) \times 10^{-6}$ cycles / radian. By comparison the expected result from the theory of Tajmar and de Matos (dashed line) is 4.81×10^{-5} cycles / radian, some 44 times larger.

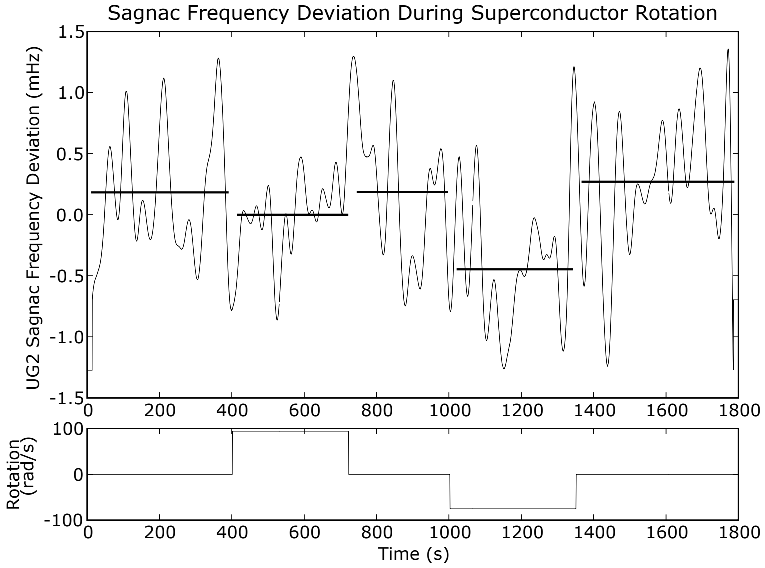

Fig. 4. Result of the frame dragging experiment. The top plot shows the measured Sagnac frequency of UG-2 with respect to time. Horizontal lines are averages for each period of rotation. The bottom plot shows the corresponding rotation of the superconductor, a positive value indicating clockwise rotation as viewed from above.

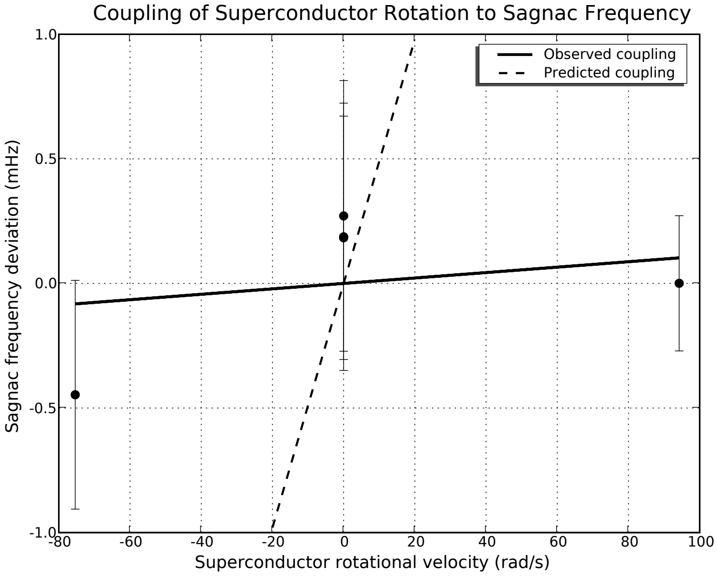

Fig. 5. Sagnac deviation of the results shown in figure 4 plotted against rotational velocity of the superconductor.

6 Conclusion

We have used UG-2, an 833.7 m^2 ring laser gyro to search for inertial frame dragging by a rotating superconductor, as predicted by Tajmar and de Matos. The noise level of UG-2 is given by microseismic disturbances and several runs were taken. This paper presents detailed results from measurements taken when these disturbances were relatively minor.

Within the uncertainty of the experiment there is no indication of inertial frame dragging due to the rotation of the nearby lead superconductor. The uncertainty of the experiment is $\sim$5% of the effect predicted [5, 6] from the theory of Tajmar and de Matos for a gravitomagnetic field anagalous to the London dipole field. We can thus place a upper limit on any frame dragging effect. If the effect exists it is at least 22 times smaller than predicted by the theory, otherwise it would have been detected in our experiment with probability $\geq$ 95%.

There have been recent claims [9] of measured effects similar to frame dragging, of strength about two orders of magnitude lower than predicted in [5,6]. These effects are speculated to be non-parity-conserving and perhaps to have a non-dipole spatial distribution at least along the rotation axis. Our experimental results do not have the sensitivity to either confirm or refute these recent claims.

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